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Technical Notes

Expression for Supersonic Fluctuating Drag Force Magnitude due to Ambient Thermodynamic Disturbances

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Nomenclature

A	=	area, L^2
C_D	=	drag coefficient
F	=	force, MLt^{-2}
g	=	gravitational acceleration, Lt^{-2}
h	=	altitude, L
M	=	Mach number
p	=	pressure, $ML^{-1}\,t^{-2}$
R	=	gas constant, $L^2\,t^{-2}\,T^{-1}$
T	=	temperature, T
u	=	streamwise velocity, Lt^{-1}
γ	=	ratio of specific heats
ρ	=	density, ML^{-3}

Subscripts and Superscripts

base	=	body base region
∞	=	far-field/ambient (freestream) quantity
$()'$	=	fluctuating quantity

Introduction

DRAG force fluctuation for free-flying aerodynamic bodies (i.e., projectiles, reentry vehicles, etc.) encountering far-field excitation [1,2], as well as self-induced disturbances [3–7], is a well-known concept that captures the interaction effects between fluid and body responses [8]. Although local fluctuation physics are not resolved, the net unsteady effect on the entire body is captured. Fluctuating drag prediction for free-flying bodies with minimal mass inertia may be important, since unsteady rigid body motion can occur. Fluctuating drag is particularly well developed for low-speed (subsonic) problems, as noted in the preceding references, but it is much less commonly discussed for high-speed (supersonic) flows [9]. An imposed, as opposed to a self induced by the body, source for disturbance is local fluctuation in the ambient temperature or density field. The presence of ambient density or temperature disturbances in the atmosphere has been characterized by many researchers [10,11]. Indeed, Field and Edwards [12] were able to demonstrate that atmospheric temperature fluctuations could be well described using stochastic process concepts. Here, we use simple fluctuation conservation (mass, momentum, and energy) expressions to derive a model for fluctuating drag magnitude (i.e., root-mean-square values) on supersonic bodies. Spectral information is not accessible within the scope of this model. The resulting expression relates mean drag to fluctuating drag and includes mean Mach number behavior that is consistent with the supersonic base pressure fluctuation data of Shvets [13]. The dependence of the drag fluctuation on the Mach number, in addition to whatever Mach variation the mean drag coefficient exhibits, implies that straightforward perturbation of the mean drag formula to obtain a drag fluctuation model is likely to be insufficient for supersonic problems.

Consider a series of far-field density or temperature disturbances denoted as

$$\left.\frac{\rho'}{\rho}\right|_{\infty}, \qquad \left.\frac{T'}{T}\right|_{\infty} \tag{1}$$

where the prime denotes a fluctuating quantity, and the unscripted is a mean value. Although denoted as far field, these conditions are considered to be behind the associated bow shock. The flow structure is, subsequently, more developed. We expect that all fluctuations are small compared with the mean values, suggesting that linearization of all equations is justifiable (i.e., any product of fluctuation quantities may be neglected). Ambient fluctuations are assumed to be in hydrostatic equilibrium, implying that state

$$p = \rho RT \rightarrow \frac{p'}{p}|_{\infty} = \frac{\rho'}{\rho}|_{\infty} + \frac{T'}{T}|_{\infty}$$

and hydrostatic equilibrium

$$p = \rho gh \rightarrow \frac{p'}{p}|_{\infty} = 0 = \frac{\rho'}{\rho}|_{\infty} + \frac{h'}{h}|_{\infty}$$

give

$$\left.\frac{\rho'}{\rho}\right|_{\infty} + \left.\frac{T'}{T}\right|_{\infty} \tag{2}$$

Recall that, although mean quantities may not be negative, fluctuations can be. Broadly, knowledge of the density or temperature in the far field gives access to the other quantity. Notice that we have restricted our model to a rather simple subset of the possible atmospheric fluctuations that a flight vehicle might encounter (i.e., those in hydrostatic equilibrium). Certainly, dynamic (velocity fluctuation driven) fluctuations are known and would require a more complete far-field model description [14].

Our interest is in the drag fluctuation associated with aerodynamic body passage through the imposed far-field disturbances. The simplest estimate for the fluctuating drag is

$$\frac{F'}{\rho u^2 A} = \frac{1}{2} C_D \left.\frac{\rho'}{\rho}\right|_{\infty} \tag{3}$$

that is, merely perturbing the density term in the drag definition by its far-field value. This type of approximation is appropriate for incompressible or low-speed flow where local flow disturbance thermodynamic energy and density coupling (energy to state) is minimal. Remarkably, Eq. (3) is also of direct use for free molecular flows, such as the drag on a satellite as it passes through the outer edge of the atmosphere [15]. In this situation, however, local continuum conservation laws simply do not hold or, more physically stated, the density is so low that the passage of the satellite causes no local disturbance and the molecules simply transfer their momentum directly to the body [16].

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However, for high-speed flow, Eq. (3) is likely to be incomplete. High-speed compressible flow requires retention of coupling between velocity, density, and temperature (i.e., satisfying mass, momentum, and energy). Fluctuation conservation equations can be written as drag definition,

$$F + F' = \tfrac{1}{2}C_D(\rho + \rho')(u + u')^2 A \tag{4}$$

mass,

$$(\rho + \rho')(u + u') = \rho u \tag{5}$$

and energy,

$$T_0 + 0 = T + T' + \frac{1}{2}T(\gamma - 1)M^2\left(1 + \frac{u'}{u}\right)^2 \tag{6}$$

The conservation expressions [i.e., Eqs. (5) and (6)] are written along a streamline at a location within the boundary layer and the boundary-layer edge. See Fig. 1 for details. Eliminating the mean and retaining first-order terms gives fluctuating drag definition,

$$\frac{F'}{\rho u^2 A} = C_D\left(\frac{1}{2}\frac{\rho'}{\rho} + \frac{u'}{u} + \cdots\right) \tag{7}$$

mass,

$$\frac{\rho'}{\rho} + \frac{u'}{u} = 0 \tag{8}$$

and energy,

$$0 = \frac{T'}{T} + (\gamma - 1)M^2\frac{u'}{u} \tag{9}$$

Notice that Eq. (6) is merely the so-called strong Reynolds analogy [17].

These expressions can be combined to eliminate ρ'/p and u'/u, providing an expression for the drag fluctuation in terms of the temperature fluctuation:

$$D' \equiv \frac{F'}{\rho u^2 A} = -\frac{1}{2}C_D\left(\frac{1}{(\gamma - 1)M^2}\right)\frac{T'}{T} \tag{10}$$

We now superimpose the ambient condition (assume that the ambient thermodynamic conditions provide forcing for the drag fluctuation model) on Eq. (10), whereby

$$D' \equiv \frac{F'}{\rho u^2 A} = -\frac{1}{2}C_D\left(\frac{1}{(\gamma - 1)M^2}\right)\frac{T'}{T}\bigg|_\infty = \frac{1}{2}C_D\left(\frac{1}{(\gamma - 1)M^2}\right)\frac{\rho'}{\rho}\bigg|_\infty \tag{11}$$

Equation (11), although very simple, provides a model for drag fluctuation magnitude due to the effect of ambient thermodynamic disturbances for high-speed flow. The physical behavior that has not been captured by the simplest estimate for fluctuation drag [i.e., Eq. (3)] is the presence of the mean Mach number term, which suggests that, as the mean Mach number increases, the fluctuating drag will decrease by M^{-2}. It is important to note that this reduction in fluctuating drag as a function of the Mach number is in addition to any Mach number effect that is likely contained in the mean drag term C_D [i.e., $C_D = C_D(M)$].

The requirement that the ambient thermodynamic conditions provide forcing for the fluctuation can be more strongly supported by writing a conservation statement between a location within the boundary layer and the ambient/hydrostatic fluctuation field (i.e., the so-called far field).

The associated expressions (see Fig. 1) are written as the momentum/Euler equation,

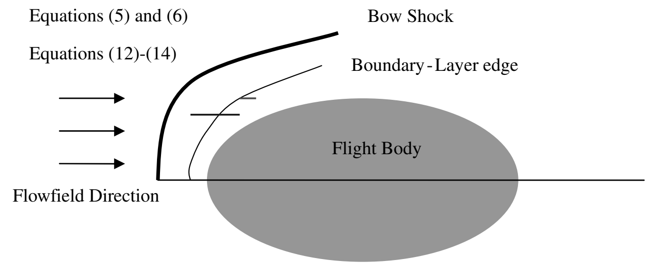

Fig. 1 Notional schematic of flight body and streamline/perturbation definitions.

$$\frac{\rho'}{\rho}\bigg|_\infty = \frac{1}{\gamma M^2}\frac{p'}{p} + \left(\frac{\rho'}{\rho} + 2\frac{u'}{u} + \cdots\right) \tag{12}$$

mass,

$$\frac{\rho'}{\rho}\bigg|_\infty = \frac{\rho'}{\rho} + \frac{u'}{u} \tag{13}$$

and energy,

$$\frac{T'}{T}\bigg|_\infty = \frac{T'}{T} + (\gamma - 1)M^2\frac{u'}{u} \tag{14}$$

For $M \gg 1$, the pressure term $(1/\gamma M^2)(p'/p)$ is small, whereby we deduce [from Eqs. (12) and (13)] that along the streamline imposed, $u'/u \ll 1$, and by Eq. (14), $T'/T|_\infty \approx T'/T$. This approximation is, of course, reflected by Eq. (11). We emphasize that Eqs. (12–14) are written between a boundary-layer location and a location beyond the boundary-layer edge; that is, the far field. Of course, passage of a high-speed vehicle induces a bow shock that may modify the strength and character of actual atmospheric fluctuations (the experiment by Keller and Merzkirch [18] discussed shock amplification density disturbances, but notes that, for large Reynolds number flows, the associated amplification is minimal). Regardless, by connecting our analysis to postshock fluctuations only, we avoid the necessity of shock amplification issues.

As suggested, fluctuating drag is virtually unknown for supersonic flow problems, making direct comparison to data difficult. Fortunately, base pressure fluctuations are well known for high-speed problems [13,19–21]. Base pressure fluctuations provide the majority of the fluctuating drag force loading, since other sources (such as boundary/shear fluctuations) are poorly spatially correlated over the vehicle length scale and cannot readily contribute to a net drag fluctuation [22]. Furthermore, base pressure fluctuations are but weakly dependent on spatial location over the body base [19], implying that one can readily map

$$F' \propto \int_{A_{\text{base}}} p'\,\mathrm{d}A$$

It is thus not surprising that measurements by both Shvets [13] and Janssen and Dutton [20,21] clearly suggest the reduction in base pressure fluctuation magnitude as the Mach number increases, as suggested by Eq. (10). Indeed, Shvets [13] offers the empirical expression for the Mach number behavior of fluctuating base pressure magnitude for conical bodies as

$$\frac{p'_{\text{base}}}{(1/2)\rho u^2} = 0.06(1 + M)^{-2} \tag{15}$$

which exhibits the same rate of decay as Eq. (10). We emphasize that the data sets described here are for self-induced drag fluctuation behavior driven by vortex shedding, separated flow unsteadiness, etc., as opposed to the fluctuation imposed by far-field disturbance, which is the focus of this research. Hence, we can but draw a scaling analogy with these results, implying that functional behavior (i.e., decay rate in terms of Mach number) may be similar, but fluctuation magnitude is certainly not comparable.

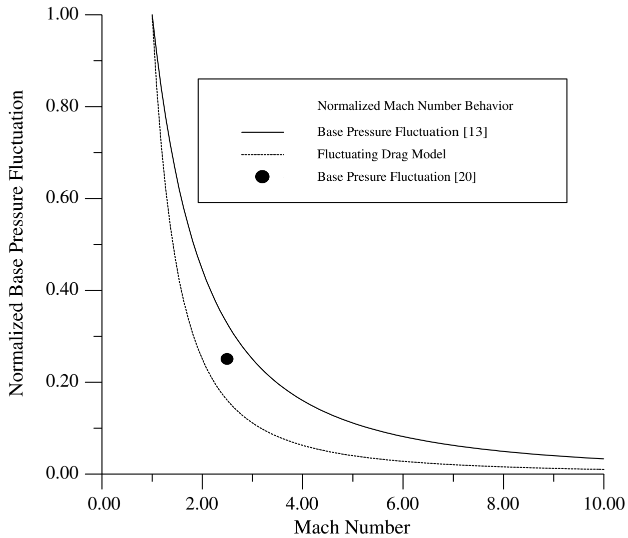

Fig. 2 Normalized base pressure fluctuation and fluctuating drag dependence on Mach number.

We can compare the functional behavior of the Shvets [13] expression for base pressure fluctuation to Eq. (10) for fluctuating drag in more detail. Since Eq. (10) is only valid for supersonic flow (recall that the strong Reynolds analogy was used, an approximate fluctuating energy equation, and is only valid for supersonic flow; it is appropriate to normalize the results to be unity for $M = 1$. Because of this normalization, the curves in Fig. 2 should only be used to understand functional behavior as opposed to magnitude. A single normalized value, as measured by Janssen and Dutton [20], for $M = 2.5$ is included. See Shvets [23] for additional data.

The results in Fig. 2 suggest the same basic dependence on the Mach number (and the same behavior for $M \to \infty$), although the drag fluctuation result derived here clearly tends to decrease more rapidly.

Conclusions

The preceding discussion suggests that high-speed fluctuating drag caused by far-field disturbance decreases with the Mach number, regardless of the mean drag compressible flow behavior. The Mach number effect for the fluctuating drag results by demanding that the fluctuation flowfield satisfy basic conservation expressions. A naïve modification of the mean drag expression does not necessarily satisfy local conservation requirements and may not provide an adequate model. By applying this more complete description, fluctuating drag (a convenient low-speed concept) can be usefully employed in a high-speed setting.

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